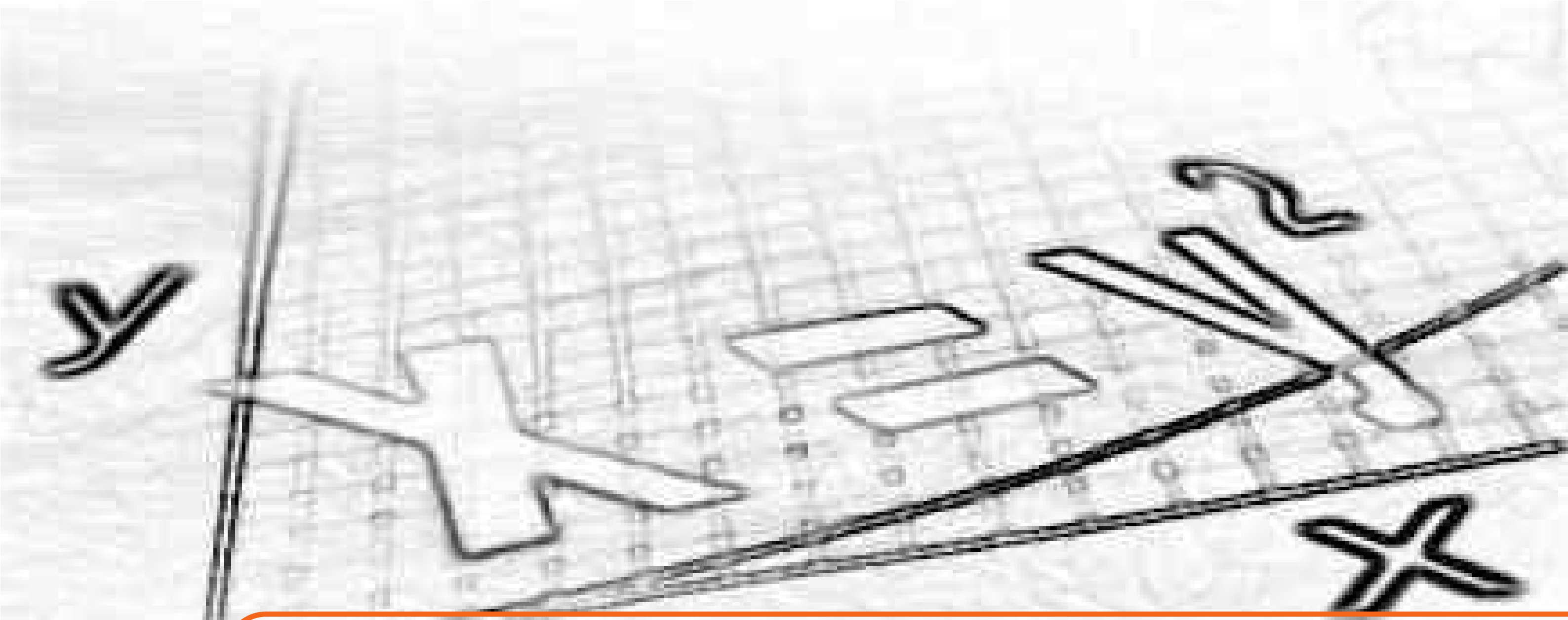


7. Systems of Equations - Substitution

7.1 Solutions of Linear Systems

We have previously discussed the idea of solving a single linear equation. We know look at the idea of systems of equations, which is essentially solving a set of equations, rather than one equation.

Definition 7.1.1 A **system of linear equations** is a set of equations in which each equation is linear.

We will be considered with systems of two equations with two variables. In order to find a solution, we need to find values for each of the variables that makes each equation in the linear system true. As an example, consider the system

$$\begin{aligned} 2x+4y &= 2 \\ 3x-2y &= -13 \end{aligned}$$

By substituting $x=-3$ and $y=2$ into the first equation, we get

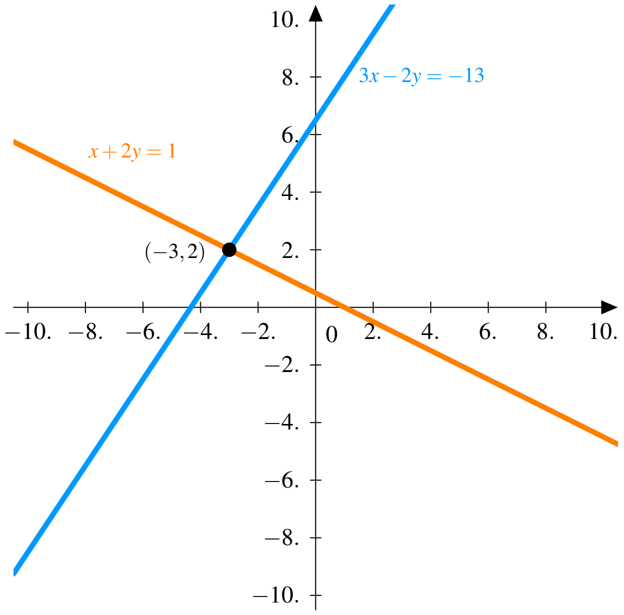
$$\begin{aligned} 2(-3)+4(2) &= 2 \\ -6+8 &= 2 \end{aligned}$$

We then substitute $x=-3$ and $y=2$ into the second equation to get

$$\begin{aligned} 3(-3)-2(2) &= -13 \\ -9-4 &= -13 \\ -13 &= -13 \end{aligned}$$

Thus, we have that $(-3,2)$ is a solution to this linear system.

We should notice that each of these equations are the equations for lines. Thus, we can graph the lines.



Since the graph is a visual representation of the solution of a linear equation, and we want to find where these two solution sets overlap, it should make sense that the intersection point is the solution.

7.2 Substitution

Since it may be difficult to find the intersection point of two graphs, we need alternative methods to solving linear systems. The first one we will consider is the substitution method.

Definition 7.2.1 The **substitution method** of solving a linear system involves solving one equation for one variable, and substituting this into the other equation.

1. **Solve for a variable**. Choose one equation, and solve for one of the variables. If possible, pick an equation/variable with a coefficient of 1 or -1 to avoid introducing unnecessary fractions.
2. **Substitute**. Substitute this into the other equation to get an equation with one variable. Then solve for this variable.
3. **Back-Substitute**. Substitute the value you just found back into the equation you found in step 1. Then solve for the remaining variable.

■ **Example 7.1** Solve the given system of equations using the substitution method.

$$\begin{aligned} x+3y&=7\\ -2x+5y&=-3 \end{aligned}$$

Solution Since the coefficient of x in the first equation is 1, we solve the first equation for x.

$$\begin{aligned} x+3y&=7\\ x&=7-3y \end{aligned}$$

We then substitute this in for x in the second equation.

$$\begin{aligned}
-2x+5y &= -3 \\
-2(7-3y)+5y &= -3 \\
-14+6y+5y &= -3 \\
-14+11y &= -3 \\
11y &= 11 \\
y &= 1
\end{aligned}$$

We then use this to substitute back into the equation we solved for x.

$$\begin{aligned}
x &= 7-3y \\
&= 7-3(1) \\
&= 7-3 \\
&= 4
\end{aligned}$$

Thus, the solution is $(4,1)$.

■

■ **Example 7.2** Solve the given system of equations using the substitution method.

$$\begin{aligned}
2x-2y &= 8 \\
4x+y &= 1
\end{aligned}$$

Solution Since the coefficient of y in the second equation is 1, we solve the second equation for y.

$$\begin{aligned}
4x+y &= 1 \\
y &= 1-4x
\end{aligned}$$

We then substitute this in for y in the first equation.

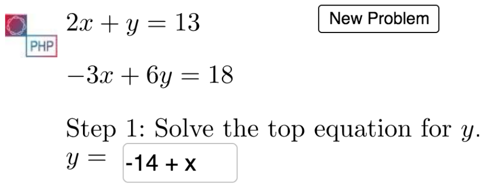
$$\begin{aligned}
2x-2y &= 8 \\
2x-2(1-4x) &= 8 \\
2x-2+8x &= 8 \\
10x-2 &= 8 \\
10x &= 10 \\
x &= 1
\end{aligned}$$

We then use this to substitute back into the equation we solved for y.

$$\begin{aligned}
y &= 1-4x \\
&= 1-4(1) \\
&= 1-4 \\
&= -3
\end{aligned}$$

Thus, the solution is $(1,-3)$.

■



Click on the image to the left to access a Geogebra applet to help you practice the substitution method. Alternatively, this applet is available at the website https://ggbm.at/ehrtgzus.