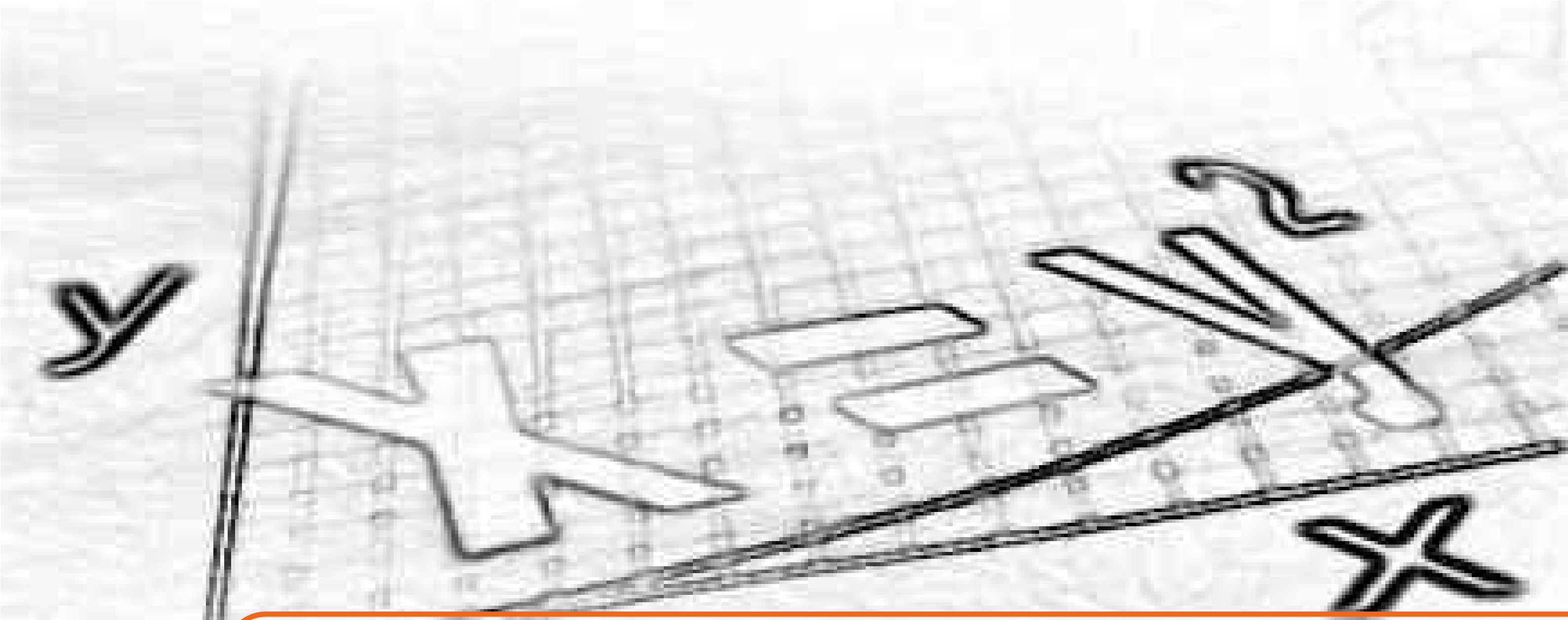
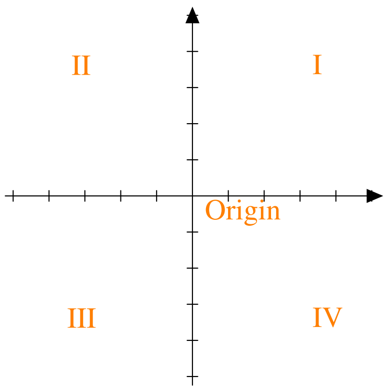


# 2. Introduction to Graphing

## 2.1 The Coordinate Plane

When dealing with numbers, we know that every real number can be plotted on a number line. When dealing with ordered pairs of numbers, we identify these with points on the coordinate plane. In order to do this, we start with two perpendicular number lines that intersect at the 0 point on each line. We call this intersection point the origin. The horizontal number line is the $x$-axis, and the vertical number line is called the $y$-axis. These axes then divide the plane into four regions, known as quadrants, shown below. We denote the quadrants with roman numerals I, II, III, and IV.



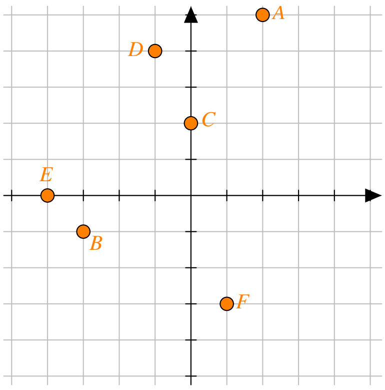

Points on this plane can be located by ordered pairs $(a, b)$. The first coordinate, or the $x$-coordinate, tells you where the point is in relation to the $x$-axis. That is, it tells you how far right or left the point is from the origin. The second coordinate, or the $y$-coordinate, tells you where the point is in relation to the $y$-axis. That is, it tells you how far up or down the point is from the origin.

As an example, consider the point $(3, -2)$. To graph this point, we would start at the origin and go right by 3 units, since the first number is three. We then go down by 2 points since the second number is $-2$. We would then have plotted this point.

■ **Example 2.1** Plot each of the following points. Then determine either which quadrant they are in, or which axis they lie on.

- $A = (2,5)$
- $B = (-3,-1)$
- $C = (0,2)$
- $D = (-1,4)$
- $E = (-4,0)$
- $F = (1,-3)$

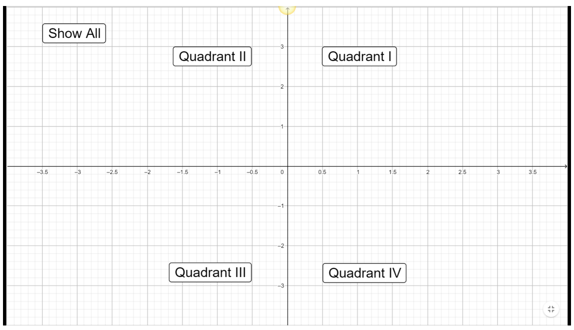
**Solution**




We also see that the points are in the following quadrants/axes:

- $A$: Quadrant I
- $B$: Quadrant III
- $C$: $y$-axis
- $D$: Quadrant II
- $E$: $x$-axis
- $F$: Quadrant IV

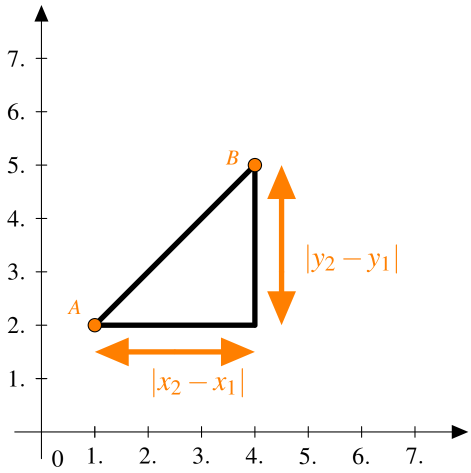
■




Click on the image to the left to access a Geogebra applet to help visualize the four quadrants. Alternatively, this applet is available at the website https://ggbm.at/nhdtpkhv.

## 2.2 Distance and Midpoint

Now that we know how to plot points, there are a few things we can determine. Let's start with how to find the distance between two given points. Consider point $A = (x_1, y_1)$ and point $B = (x_2, y_2)$. We will denote the distance between these two points $d(A,B)$. Let's see if we can determine the distance between these two points. Start by drawing the right triangle whose hypotenuse is given by the line between $A$ and $B$. We can then use the fact that the distance between two numbers $a$ and $b$ is given by $|b-a|$ to find the lengths of the legs of this triangle, as seen in the diagram below.




We can then use the Pythagorean theorem to get the length of the hypotenuse, which is $d(A,B)$:

$$(|x_2 - x_1|)^2 + (|y_2 - y_1|)^2 = d(A,B)^2$$

Solving for $d(A,B)$ then gives

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Definition 2.2.1** Given two points $A = (x_1, y_1)$ and $B = (x_2, y_2)$, the distance between them, denoted by $d(A,B)$ is given by

$$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

■ **Example 2.2** Find the distance between the points $A = (3,-2)$ and $B = (-1,5)$.

**Solution** In this case, we have $x_1 = 3$, $y_1 = -2$, $x_2 = -1$, and $y_2 = 5$. Substituting these into the formula gives

$$\begin{aligned} d(A,B) &= \sqrt{(-1-3)^2 + (5-(-2))^2} \\ &= \sqrt{(-4)^2 + (7)^2} \\ &= \sqrt{16+49} \\ &= \sqrt{65} \end{aligned}$$

■

The next thing we want to consider involves the line segment with endpoints $A$ and $B$. We want to find the midpoint of this line segment. That is, we want to find the point exactly in the middle of $A$ and $B$. Let's consider only the $x$-coordinate. Suppose $A = (x_1, y_1)$ and $B = (x_2, y_2)$. We can then find the midpoint by "averaging" these two points.

**Definition 2.2.2** Given two points $A = (x_1, y_1)$ and $B = (x_2, y_2)$, the midpoint of the line segment with endpoints $A$ and $B$ is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

■ **Example 2.3** Find the midpoint of the line segment with endpoints $A = (3,-2)$ and $B = (-1,5)$.

**Solution** In this case, we have $x_1 = 3$, $y_1 = -2$, $x_2 = -1$, and $y_2 = 5$. Substituting these into the formula gives

$$\left(\frac{3+(-1)}{2}, \frac{-2+5}{2}\right)$$
$$\left(\frac{2}{2}, \frac{3}{2}\right)$$
$$(1, 1.5)$$

■

## 2.3 Graphing Equations

Now that we are a little more comfortable with points and the coordinate plane, let's look at how to graph an equation. There are several ways to do this, but for now, we will look at the point-plotting method. For this, we need an equation in two variables, $x$ and $y$. The graph of the equation is a visual solution to this equation. That is, every point on the graph makes the equation true, and every ordered pair that makes the equation true is on the graph.

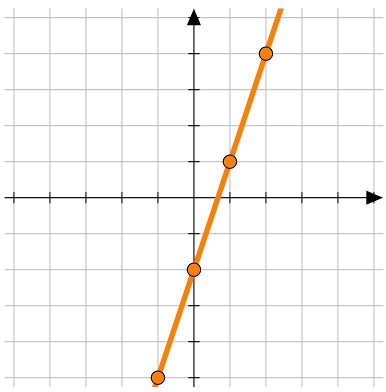
For the point-plotting method, we just pick a few values for $x$ to be, and then determine what $y$ needs to be. We can then plot the points and connect the dots with a smooth curve. Let's demonstrate this with a few examples.

■ **Example 2.4** Graph the equation $y = 3x - 2$ by letting $x = -2, -1, 0, 1, 2$.
**Solution** We start by substituting these values in for $x$ to determine $y$:

$$\begin{aligned}
x = -2: &\quad y = 3(-2) - 2 = -8 \\
x = -1: &\quad y = 3(-1) - 2 = -5 \\
x = 0: &\quad y = 3(0) - 2 = -2 \\
x = 1: &\quad y = 3(1) - 2 = 1 \\
x = 2: &\quad y = 3(2) - 2 = 4
\end{aligned}$$

This is typically displayed in table form (as seen below). We can then plot these point, and connect them with a line:

| $x$ | $y = 3x - 2$ |
|---|---|
| $-2$ | $-8$ |
| $-1$ | $-5$ |
| $0$ | $-2$ |
| $1$ | $1$ |
| $2$ | $4$ |



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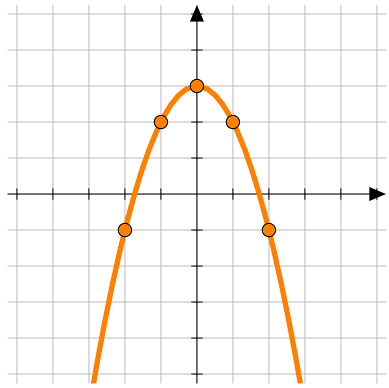
■ **Example 2.5** Graph the equation $y = -x^2 + 3$ by letting $x = -2, -1, 0, 1, 2$.

**Solution** We start by substituting these values in for $x$ to determine $y$:

$$\begin{aligned}
x = -2: &\qquad y = -(-2)^2 + 3 = -4 + 3 = -1 \\
x = -1: &\qquad y = -(-1)^2 + 3 = -1 + 3 = 2 \\
x = 0: &\qquad y = -(0) + 3 = 0 + 3 = 3 \\
x = 1: &\qquad y = -(1)^2 + 3 = -1 + 3 = 2 \\
x = 2: &\qquad y = -(2)^2 + 3 = -4 + 3 = -1
\end{aligned}$$

This is typically displayed in table form (as seen below). We can then plot these point, and connect them with a line:

| $x$ | $y = -x^2 + 3$ |
|---|---|
| $-2$ | $-1$ |
| $-1$ | $2$ |
| $0$ | $3$ |
| $1$ | $2$ |
| $2$ | $-1$ |



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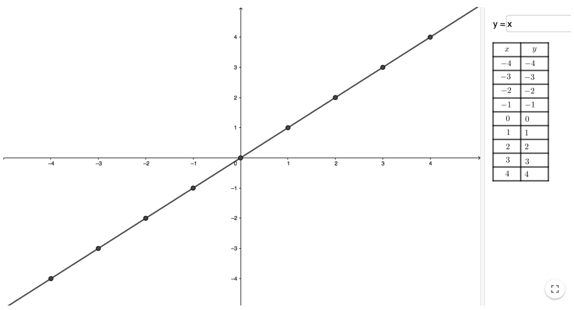


Click on the image to the left to access a Geogebra applet to help practice the point-plotting method. Alternatively, this applet is available at the website https://ggbm.at/xymqyvhm.

## 2.4 Intercepts

There is a lot of information we can get from graphs. The first one that we will look at is the intercepts. There are two types of intercepts: $x$-intercepts and $y$-intercepts. Simply put, intercepts are where the graph crosses the axes.

The $x$-intercept(s) of a graph is where the graph crosses the $x$-axis. Since this point lies on the $x$-axis, we know that the $y$-coordinate is 0. Thus, $x$-intercepts are always of the form $(a, 0)$. This means we can find the $x$-intercepts of an equation by letting $y = 0$ and solving for $x$.

Similarly, the $y$-intercept of a graph is where the graph crosses the $y$-axis. Since this point lies on the $y$-axis, we know that the $x$-coordinate is 0. Thus, $y$-intercepts are always of the form $(0, b)$. This means we can find the $y$-intercepts of an equation by letting $x = 0$ and solving for $y$.

■ **Example 2.6** Find the $x$- and $y$-intercepts of $y = 3x - 2$.

**Solution** In order to find the $x$-intercept, we need to let $y = 0$.

$$\begin{aligned} 0 &= 3x - 2 \\ +2 & \qquad +2 \\ 2 &= 3x \\ \frac{2}{3} &= \frac{3x}{3} \\ \frac{2}{3} &= x \end{aligned}$$

Thus, the $x$-intercept is $(2/3, 0)$. To find the $y$-intercept, we need to let $x = 0$.

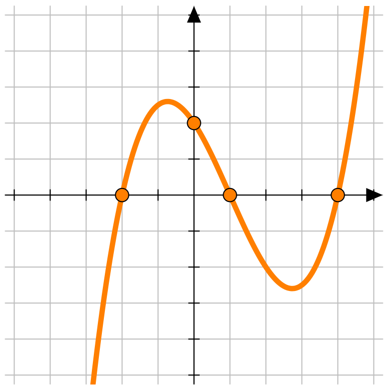
$$\begin{aligned} y &= 3(0) - 2 \\ y &= -2 \end{aligned}$$

Thus, the $y$-intercept is $(0, -2)$.

■

■ **Example 2.7** Find the $x$- and $y$-intercepts of the equation whose graph is given below.

**Solution**



The $x$-intercepts occur when the graph crosses the $x$-axis. We can see this happens at $x = -2$, $x = 1$, and $x = 4$. Thus, the $x$-intercepts are $(-2, 0)$, $(1, 0)$, and $(4, 0)$. The $y$-intercepts occur when the graph crosses the $y$-axis. We can see this happens when $y = 2$. Thus, the $y$-intercept is $(0, 2)$.

■